Stripe patterns in a granular system induced by slow deformation of its container

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We investigate the formation of stripe patterns that appear on the surface of a dry granular system as the container is deformed very slowly. In an experimental study using nearly mono-disperse glass beads, we found that many faults develop beneath the surface. Our results show that the spacing of stripes is independent of the system size and does not depend significantly on the grain size.

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Granular systems under the influence of gravity exhibit a rich variety of interesting phenomena [1–3]. In the solid phase, it is well known that granular systems differ greatly from ordinary solids with regard to statistical properties of stresses and that the stress distributions within them depend strongly on the history of their formation [4–6]. Also the deformation of granular matter is usually accompanied by sheet-like localized fluidization appearing in an otherwise solid phase. Such a fluidized region is called a "shear band", and if it is formed in the bulk, rather than the surface, it is called a "fault" [7–9].

We experimentally investigated the formation of stripe patterns formed on the surface of a system of granular matter as a result of the slow deformation of its container [10]. We used spherical glass beads with a nearly mono-disperse size distribution. Our experiments show that these stripes were caused by the formation of many faults beneath the surface, not fluidization on the surface.

Although our investigations were inspired by the accidental finding of one of authors(Kurumatani), we recently realized that in 1928 T. Terada and N. Miyabe reported similar phenomena in the formation of a series of faults in dry sand [11]. They are pointed out similarities with certain geological phenomena. They also found that the faults are approximately parallel and that the angle of inclination of the faults depends on the order of packing. The behavior of these faults reminds us of the formation and arrangement of shear bands in rocks, which have been investigated by several people [12]. The mechanism responsible for the arrangement of faults in granular systems has not yet been revealed. Here we report experimental results elucidating the dependence on various conditions of the spacing of stripes created by faults.

We prepared three kinds of spherical glass beads, made of soda-lime silicated glass of density $2.5 \pm 0.1 g/cm^3$, with average diameters $D = 40, 100$ and $200 \mu m$. We refer to them as GB40, GB100 and GB200. Their size distributions were nearly mono-disperse, with $80 - 90\%$ of the particles having diameters between $0.9D$ and $1.1D$ [13]. We used a fourth kind of glass beads, with 0.1 mm diameter (GB-01), only for experiments in which we prepared colored layers of beads. The size distribution of GB-01 was somewhat less mono-disperse than those of the other beads.

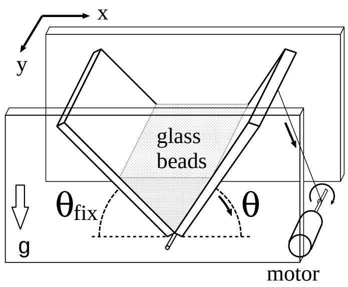

FIG. 1. The experimental setup.

A schematic depiction of the container and glass beads used in our experiments is given in Fig. 1. As shown there, two rectangular plates are joined to form a V-shape structure held between parallel lateral walls. We refer to the plates as "V-walls". V-walls and the lateral walls are all made of acrylic plates and V-walls are covered with thin rubber sheets to protect the acrylic from being scratched. The slope of each V-wall is variable. In an experiment, we fixed one of the plates at an angle θ_{fix} and decreased the angle of the other plate, θ, from some initial angle θ_0 at an almost constant angular velocity, ω. For this purpose, we used a motor connected to a power supply with a constant voltage to pull this plate with a string. We first poured a single kind of beads with total weight Mg into the container and tapped it a few times to level the surface horizontally. For each experiment, the volume fraction of glass beads in the container was $0.53 - 0.61$. The volume fraction depended slightly on the type of beads used, and it was not exactly the same for each experiment. However, the results described below seem to be largely insensitive to these differences.

We consider the case with GB100 beads and with parameter values $M = 400g$, $\theta = 30°$, $\theta_0 = 60°$, $\omega = 0.60 \pm 0.05°/s$ and $\theta_{fix} = 50°$ as the 'standard'. In the results described in this paper, we give explicit parameter values only when they differ from these. In the actual experiments, the above-described setup was placed into an airtight container. Before carrying out the experiments, we dried the beads by placing them in

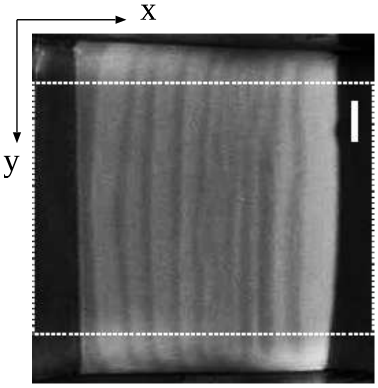

FIG. 2. A snapshot of a stripe pattern for $\theta = 30^\circ$.

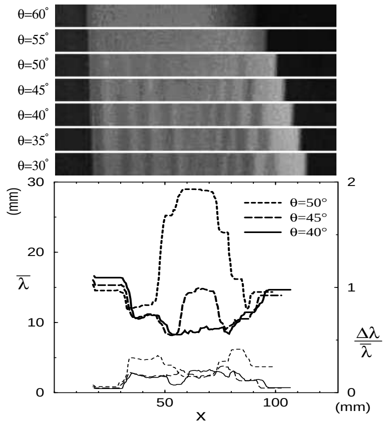

FIG. 3. The process of the formation of stripes.

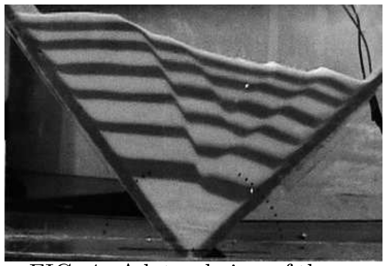

FIG. 4. A lateral view of the container at $\theta = 45^\circ$ in the case that horizontal colored layers were prepared initially.

this container and using dessicants to keep the relative humidity in the range $5-15\%$. This was done for several days, until their total weight no longer decreased with time.

Lighting the surface obliquely to make the unevenness clearly visible, we photographed the system from a position directly above, using a digital video recorder. Figure 2 displays a typical gray-scale image of a stripe pattern, where the width of the image corresponds to a length of $128.3mm$ on the surface, and the vertical axis y is parallel to the rotation axis of V-walls. It is seen that stripes extend along the entire length of the granular surface, except near the lateral walls, and that they are almost parallel to the y axis. We also see that the spacing of the stripes depends significantly on the value of the x coordinate. In order to calculate the spacing of the stripes $\lambda(x,y)$ at the place (x,y) as a function of position, we processed a image as follows.

We cropped the central region of a snapshot, as shown in Fig. 2 with dotted lines, and measured the brightness b_{ij} at a set of points $\{(i,j)\}$ of the cropped digital image, where $x = i\Delta x$, $y = j\Delta x$ and $\Delta x = 0.513mm$. We first defined the left boundary as $i = l(j)$ and the right boundary as $i = r(j)$ of the granular surface between V-walls by setting some threshold for $\langle b_{ij}\rangle$, where $\langle\cdot\rangle$ represents an average over the rectangular region of size 5×31 pixels centered at (i,j). The small white rectangle in Fig. 2 displays this size. We next calculated the gradient of b_{ij} with respect to i as $s_{ij} \equiv \left(\langle ib_{ij}\rangle - \langle i\rangle\langle b_{ij}\rangle\right) / \left(\langle i^2\rangle - \langle i\rangle^2\right)$, using a least square method. For each value of j, we determined the intervals in which s_{ij} takes negative values, and for the kth such interval of decreasing brightness, $I_j^{(k)}$, we calculated the 'increment of brightness' $\Delta s_j^{(k)} \equiv \sum_{i\in I_j^{(k)}} s_{ij}$ and the average of s_{ij}, $\overline{s}_j^{(k)}$. We defined the right edge of any interval $I_j^{(k)}$ that satisfies both conditions $\Delta s_j^{(k)} < -0.005$ and $\overline{s}_j^{(k)} < -0.0002$ to be the center of a dark stripe. The value 0.005 is on the order of the smallest difference in brightness that can be distinguished from the digital images.

We denote to the centers of dark stripes as $i = c_j^{(m)}$, where $m = 1,2,...,m_j$ and $c_j^m < c_j^{(m+1)}$, and the boundaries of the granular surface as $c_j^{(0)} \equiv l(j)$ and $c_j^{(m_j+1)} \equiv r(j)$. For given $y = \Delta xj$, the spacing of stripes $\lambda(x,y)$ is defined as the distance $\Delta x(c_j^{(m+1)} - c_j^{(m)})$, where $\Delta xc_j^{(m)} \le x < \Delta xc_j^{(m+1)}$. Using $\lambda(x,y)$ defined in this manner, at each x we determined its average over y and over five experiments, $\overline{\lambda}(x)$, and the corresponding standard deviation, $\Delta\lambda(x)$. We display these functions in Fig. 3, where the thick curves represent $\overline{\lambda}(x)$, with the ordinate scale appearing on the left and the thin curves represent $\Delta\lambda(x)/\overline{\lambda}(x)$, with the ordinate scale appearing on the right. We find that the quantity $\Delta\lambda/\overline{\lambda}$ is in the range $0.1 - 0.3$ for uniformly aligned stripes, while it takes larger values for disordered or branching stripes. Above the graph appear parts of snapshots cropped from the central region with respect to y. The x coordinates correspond to the abscissa of the graph below.

We describe the experimental results below.

Figure 3 displays a series of snapshots taken during the formation of a stripe pattern from $\theta_0 = 60^\circ$ to $\theta = 30^\circ$ at decrements of 5°, and data for $\theta = 50^\circ, 45^\circ$ and 40°. We find that stripes begin to form before $\theta_0 - \theta$ reaches 10°, and their amplitudes increase as this value increases. We also find that stripes near centered values of x are observed later than those near V-walls. Also, the spacing of stripes decreases when moving from either V-wall forward the center, along the x axis.

We were able to observe the movement of grains at the lateral walls because the lateral walls are made of transparent acrylic. In order to visualize the displacement of

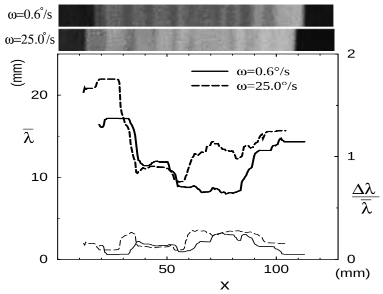

FIG. 5. Dependence on the angular velocity.

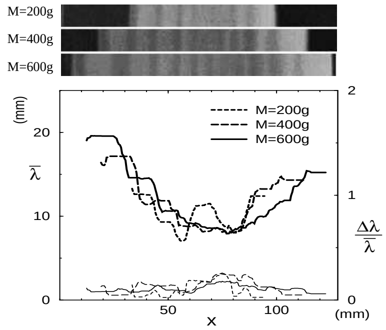

FIG. 6. Dependence on the system size.

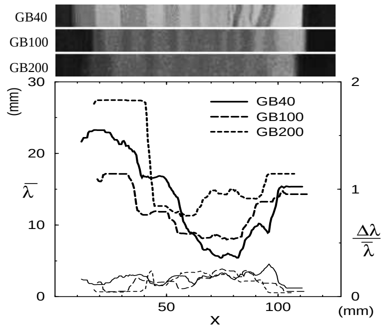

FIG. 7. Dependence on the grain size.

grains, we prepared horizontal layers of different colored beads prior to the rotation of the plate. In this case we used a larger container, with a setup essentially the same as that described in Fig. 1. In this set of experiments, we used the glass beads GB-01 with $M = 1000g$. The beads were stained with a water-soluble ink. According to our comparison of systems of stained and unstained beads, there is no difference between the behavior of the two. Figure 4 displays a lateral view of the container. Several faults are observed on each side of the V-shaped container. We find that each fault appears as a stripe on the surface. Also, apparently, every sufficiently thick stripe corresponds to some fault, although it was not possible to detect faults corresponding to narrow stripes appearing near the central region with respect to x. We believe that this is due to disturbance of the lateral boundaries. The faults we observed were found to be nearly straight, although long faults exhibited slight curves. In our experiment, the most distinct fault, which appeared near the fixed plate, made an angle of $63 \pm 2°$ with respect to the horizontal. We note that the fault was not parallel to the fixed plate with $\theta_{fix} = 50°$. While faults near the moving plate were formed at approximately the same angle (inclined on the opossite direction), their slopes decreased as the plate was rotated.

When we set the angle of the fixed plate θ_{fix} at smaller values, we found that no stripe was formed in the region near the fixed plate, where the surface remains almost horizontal. While the size of this region increased as θ_{fix} decreased, it was found that neither the width of the region in which stripes appear nor the spacing of stripes depends on θ_{fix}. These independences suggest that the inclination of the faults we measured above was determined independently on θ_{fix}.

We also conducted experiments in an air environment with higher humidity($\sim 50\%$) and found that striped patterns appear in this case too. To test the case of very low humidity, we also carried out experiments in a nitrogen environment. Before the experiments, we filled the airtight container with nitrogen and then flushed it several times, until the weight of the glass beads no longer decreased. In comparing the results obtained in this case and that described in Fig. 3, we were able to find no difference in the striped patterns appearing in each, within experimental error. In the experiments of Terada and Miyabe mentioned above, they investigated a granular system consisting of baked sand. Our and their results together provide evidence that stripe patterns may appear generally in completely dried granular systems.

We display the results for significantly different angular velocities $\omega = 0.6$ and $25.0°/s$ in Fig. 5. We find that the spacing of stripes changes little as ω is increased, although the stripes become somewhat less pronounced. We found that the case in which $\omega = 0.6°/s$ is essentially quasi-static. Thus our results show that stripe patterns appear in the quasi-static deformation of a container and that the spacing is essentially independent on the deformation speed.

Figure 6 displays the data for experiments with different total weights of beads, $M = 200, 400$ and 600, for which the dimension of the system along the x direction increases in proportion to $\sqrt{M}$. The three functions $\overline{\lambda}(x)$ are identical, within experimental error. We find that the spacing of stripes is almost independent of the system size.

In Figure 7, we display the results for the three kinds of glass beads GB40, GB100 and GB200. In the case of GB40, the boundaries between the stripes are sharp, and we observe fine stripes in the central region. By contrast, stripes in the central region are difficult to discern in the case of GB200, which consists of grains with 5 times larger diameter than GB40. Except in the central region, we found that the spacing of stripes does not depend significantly on the size of grains. It is, however, difficult to determine the exact dependence on the grain size because the size distributions and the initial volume fractions of beads in the container differ slightly for the different kinds of beads.

As described above, the stripe patterns observed in our experiments result from the formation of faults developing throughout the bulk of the system, not from surface flow. It is well known that a typical shear band in a granular system has a thickness on the order of several grain diameters, and it is thought to increase with the shear velocity [2,9,7]. We believe that these dependences are the reason that the stripes become less pronounced as either the size of the beads or the rate of deformation of the container increases.

It is believed that a shear band in a dry granular system created in the plane in which the Mohr-Coulomb criterion $\sigma_t \geq \tan\phi_c\, \sigma_n$ is first satisfied, where σ_n and σ_t represent the normal and shear components of the stress on this plane. The constant ϕ_c is called the 'maximum angle of internal friction'. By regarding the system as a two-dimensional continuous medium and assuming that this criterion holds critically [i.e. $\sigma_t = \tan\phi_c\, \sigma_n$] at a fault, the inclination of the faults ϕ is given by $\phi = \phi_c/2 + 45°$ with respect to the minimum principal axis of the stress [2,9,11]. In our system, the above-described results show that the faults formed near the beginning of the rotation, at a time when the granular surface was still nearly horizontal. If we assume that the minimum principal axis was parallel to this granular surface, we can regard the value of ϕ given above to be measured with respect to the horizontal. ϕ_c is usually considered, as an approximation, the maximum angle of the slope at which there occurs no surface flow. Measuring this maximum angle for GB-01 to be $28.1{\pm}0.7°$, we estimate that the angle of the faults should be $\phi = 59.0 \pm 0.4°$. While this is not consistent with the experimental value $63 \pm 2°$, these values are quite close, and thus it appears that the assumptions made above are reasonable as the first approximation.

As stated above, the spacing of stripes does not depend significantly on either the system size or the grain size, and the phenomena we have studied are induced by quasi-static deformation. In dry granular systems consisting of spherical rigid particles, frictional coefficients and the acceleration due to gravity are the only parameters characterizing the forces involved in the quasi-static dynamics. A simple dimensional analysis shows that such systems have no characteristic length other than the system size and the grain size. For this reason, we believe that the mechanism responsible for arranging faults possesses no characteristic length to determine their spacing. We note that this differs from the situation involving shear bands appearing in rocks [12]. Our finding that the spacing of stripes becomes smaller as x approaches the central region leads us to believe that the spacing in a particular region depends on the depth of the faults in this region that create the stripes.

In summary, we have reported experimental results for a granular system in which stripe patterns result from the slow deformation of a V-shaped container. Our results show that this behavior occur both in the dry granular systems and in quasi-static deformation. The stripe patterns are caused by the formation of many faults, which are arranged approximately parallel to one another on each side of the V-shaped container. From the measurement of the inclination of the faults, we conclude that the stress on each faults is approximately critical at the time that the stripes begin to form. We also found that the spacing of stripes becomes smaller in the central region, away from V-walls, and depends significantly on neither the system size nor the grain size. These results suggest that the mechanism responsible for arranging the faults possesses no characteristic length to determine the spacing.

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